Fachbereich Mathematik der Universität Hamburg **WiSe 2021/22**
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Analysis III for Engineering Students

Work sheet 6

Exercise 1:

a) Given are the below described body $K \subset \mathbb{R}^3$ and the vector field $\boldsymbol{f}$:

$$K = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \;\middle|\; x \in [1,3],\;\; 1 - x \leq y \leq 2 + x,\;\; x^2 + y^2 - 1 \leq z \leq x^2 + y^2 + 1 \right\},$$

$$\boldsymbol{f} : \mathbb{R}^3 \to \mathbb{R}^3, \qquad \boldsymbol{f}(x,y,z) = \begin{pmatrix} yz + y \\ x(z+1) + y \\ y(z+2) + x \end{pmatrix}.$$

Compute

$$\int_K \operatorname{div}(\boldsymbol{f}(x,y,z))\, \mathrm{d}(x,y,z).$$

b) Compute the integral

$$\int_D (1 - x^2) d(x,y)$$

over the annulus

$$D := \left\{ (x,y)^T \in \mathbb{R}^2;\; 1 \leq x^2 + y^2 \leq 4 \right\}.$$

Hint: $\cos^2 \phi = \frac{1}{2}(\cos(2\phi) + 1)$.

Exercise 2:

Given the cone $K \subset \mathbb{R}^3$, $K = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} : 0 \leq x^2 + y^2 \leq 1,\, 0 \leq z \leq 4 - 4\sqrt{x^2 + y^2} \right\}$.
The cone has the constant density $\rho = 2$.

a) Compute the mass of the cone.

b) Compute the moment of inertia of the cone with respect to the $z-$ axis using integration.

c) Compute the moment of inertia of the cone with respect to an axis A, parallel to the $z-$ axis, passing through the point $(\frac{3}{2}, 0, 0)^T$.

Discussion: 10.01–14.01.22