

Analysis III for Engineering Students Homework sheet 7

Exercise 1:

Given vector fields $\mathbf{f}, \mathbf{g} : \mathbb{R}^3 \rightarrow \mathbb{R}^3$,

$$\mathbf{f} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2xz \\ -2yz \\ x^2 - y^2 \end{pmatrix} \quad \text{und} \quad \mathbf{g} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x^2 + z \\ y^2 z + z^3 \\ -y \end{pmatrix}$$

- a) Compute the potentials of \mathbf{f} and \mathbf{g} , if it is possible.
- b) Given

$$\mathbf{c} : \left[0, \frac{\pi}{6}\right] \rightarrow \mathbb{R}^3, \quad \mathbf{c}(t) = \begin{pmatrix} t \\ \cos(3t) \\ \sin(3t) \end{pmatrix}.$$

Compute the line integrals

$$\int_{\mathbf{c}} \mathbf{f} d\mathbf{x}, \quad \text{and} \quad \int_{\mathbf{c}} \mathbf{g} d\mathbf{x}.$$

Exercise 2:

Given the body $K := \{\mathbf{x} \in \mathbb{R}^3 \mid x^2 + y^2 \leq 4, 0 \leq z \leq 5 - x + y, \}$
and the vector field $\mathbf{f}(\mathbf{x}) := (xz, yz, xyz)^T$.

- a) Sketch the body K and provide the parametrizations for the three smooth surfaces F_1, F_2 and F_3 , which bound K .
- b) Compute the volume integral $\int_K \operatorname{div} \mathbf{f} d\mathbf{x}$.
- c) Compute the flow of \mathbf{f} through the surfaces F_1, F_2 and F_3

Submission deadline: 24.01.–28.01.22