Fachbereich Mathematik der Universität Hamburg **WiSe 2021/22**
Prof. Dr. I. Gasser
Dr. H. P. Kiani, S. Onyshkevych

Analysis III for Engineering Students

Work sheet 7

Exercise 1:

a) Let $\boldsymbol{f}$ be the vector field $\boldsymbol{f}(x,y) = \begin{pmatrix} x^2 \\ y^2 \end{pmatrix}$, $\mathbf{c}_1$ be the curve with the parametrization

$$\mathbf{c}_1(t) = (t, \sin(t)) \qquad t \in [0, \pi]$$

and c_2 be the mathematically positive-oriented edge of the rectangle

$$R = \{(x,y) \,:\, x \in [0,1],\ y \in [0,2]\} \;=\; [0,1] \times [0,2]\,.$$

(i) Does vector field f have potential?

(ii) For $i = 1, 2$ compute the line integrals

$$\int\limits_{\mathbf{c}_i} \boldsymbol{f}(x,y)\, d(x,y)\,.$$

(iii) Compute the flow of $\boldsymbol{f}$ through R.

b) Let $\tilde{\boldsymbol{f}}$ be the vector field $\tilde{\boldsymbol{f}}(x,y) = \begin{pmatrix} x^2 - y^3 \\ y^2 + x^3 \end{pmatrix}$, $\mathbf{c}_2$ be defined as above and

$$\mathbf{c}_3(t) = (1, t^2) \qquad t \in [0,3].$$

Compute the line integrals

$$\int\limits_{\mathbf{c}_2} \tilde{\boldsymbol{f}}(x,y)\, d(x,y)\,, \qquad \int\limits_{\mathbf{c}_3} \tilde{\boldsymbol{f}}(x,y)\, d(x,y)\,.$$

Exercise 2)

Given are

$$K := \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \,:\, 0 \leq x^2 + y^2 + z^2 \leq 16,\, y \geq 0 \right\},$$

and the vector field

$$\boldsymbol{f} : \mathbb{R}^3 \to \mathbb{R}^3,\ \boldsymbol{f}(x,y,z) = \begin{pmatrix} x + y^2 \\ 2y \\ 3z + x^2 \end{pmatrix}.$$

a) Compute the $\displaystyle\int_K \operatorname{div}\, \boldsymbol{f}(x,y,z)\, d(x,y,z)$.

b) K is bounded by a flat surface W and a curved surface M. Provide the parametrization of W.

c) Compute the flow of $\boldsymbol{f}$ through W, i.e

$$\int_W \boldsymbol{f} \cdot dO\,.$$

d) According to a) and c), how large is the flow through the curved part of the edges of K, i.e

$$\int_M \boldsymbol{f} \cdot dO\,?$$

Discussion: 24.01–26.01.22