26.10.09

$$y'(t) + a(t)\,y(t) = h(t)$$

$$y_p'(t) + a(t)\,y_p(t) = h(t) \qquad y_p \text{ Partikulärl.}$$

$$(y-y_p)'(t) + a(t)(y-y_p)(t) = 0$$

$$y_h = y - y_p \quad \text{löst homogene Gl.}$$

$$y' = f(y,t)$$

$$y_p' = f(y_p,t)$$

$$(y-y_p)' = f(y,t) - f(y_p,t) \neq f(y-y_p,t)$$

$$c'(t) = h(t) \exp\left\{ + \int_{t_0}^{t} a(\tau)\, d\tau \right\}$$

$$c(t) = \int_{t_0}^{t} h(\tau) \exp\left\{ + \int_{t_0}^{\tau} a(s)\, ds \right\} d\tau$$

$$u' = -tu + t$$

a) $u_h = c\, e^{-\frac{t^2}{2}}$

$$\int \frac{du_h}{u_h} = -\int t\, dt = -\frac{t^2}{2}$$

$\ln u_h$

$$u_p = c(t)\, e^{-\frac{t^2}{2}} = \dots\dots = \underline{\int_{t_0}^{t} \tau\, e^{-\frac{\tau^2}{2}}\, d\tau} \cdot e^{-\frac{t^2}{2}} = \dots$$

b) $u' = -tu + t = -t(u-1)$

$v = u - 1, \quad v' = u'$

$v' = -tv$

$v = c\, e^{-\frac{t^2}{2}} \Rightarrow u = 1 + c\, e^{-\frac{t^2}{2}}$

$y = 1 + \frac{1}{u} = 1 + \frac{1}{1 + c\, e^{-\frac{t^2}{2}}}$

c) $u' = -tu + t$

$u_h = c\, e^{-\frac{t^2}{2}}$ $\qquad u_p = a + bt$

$\ldots$

$$g(t,y) + h(t,y)\,y' = 0$$

An $\exists\, \phi = \phi(t,y)$

$$\frac{\partial \phi}{\partial t} = g \qquad \frac{\partial \phi}{\partial y} = h$$

$$\underline{\frac{\partial^2 \phi}{\partial y \partial t}} = \frac{\partial g}{\partial y} \overset{?}{=} \frac{\partial h}{\partial t} = \underline{\frac{\partial^2 \phi}{\partial t \partial y}}$$

falls $\phi \in C^2 \Rightarrow$ "$=$"

falls g und $h \in C^1 \Rightarrow$ "$=$"

also, überprüfe $\frac{\partial g}{\partial y} \overset{?}{=} \frac{\partial h}{\partial t}$ falls ja $\Rightarrow \exists\, \phi$

$$\underbrace{(1+2ty+y^2)}_{g} + \underbrace{(t^2+2ty)}_{h}\,y' = 0$$

$$\frac{\partial g}{\partial y} = 2t+2y = 2t+2y = \frac{\partial h}{\partial t}$$

$\Rightarrow \exists\, \phi$

$$\frac{\partial \phi}{\partial t} = g \quad \Rightarrow \phi = \int (1+2ty+y^2)\,dt$$

$$= t + t^2 y + t y^2 + c(y)$$

$$\frac{\partial \phi}{\partial y} = \underline{t^2 + 2ty + c'} = h = \underline{t^2+2ty}$$

$\Rightarrow c' = 0 \Rightarrow c = \text{const}$

$$\Rightarrow \phi = t + t^2 y + t y^2 + c = 0$$

$$y = \ldots$$