Fachbereich Mathematik der Universität Hamburg **Winter 2021/22**
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Differential Equations I for Students of Engineering Sciences

Sheet 3 (in-class)

Exercise 1:

Consider the following initial value problem for $t \neq 0$:

$$\begin{aligned} \dot{y}_1 &= y_2 \\ \dot{y}_2 &= 3y_1/t^2 + y_2/t \end{aligned} \qquad \text{with} \quad y_1(1) = 0 \text{ and } y_2(1) = 4.$$

a) Express the initial value problem in terms of matrices and vectors using the notation $\boldsymbol{y}(t) = (y_1(t), y_2(t))^T$.

b) Determine a polynomial solution of the form

$$\boldsymbol{y}^1(t) = \begin{pmatrix} a_0 + a_1\, t + a_2\, t^2 + a_3\, t^3 \\ b_0 + b_1\, t + b_2\, t^2 + b_3\, t^3 \end{pmatrix}.$$

c) Do $\boldsymbol{y}^1(t)$ and $\boldsymbol{y}^2(t) := \begin{pmatrix} 1/t \\ -1/t^2 \end{pmatrix}$ form a fundamental system of the system of differential equations?

d) Solve the initial value problem.

Exercise 2:

Compute the general solution of the following system of differential equations

$$\boldsymbol{y}' = \begin{pmatrix} 7 & -6 \\ -6 & -2 \end{pmatrix} \boldsymbol{y}.$$

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