Fachbereich Mathematik der Universität Hamburg **Winter 2021/22**
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Differential Equations I for Students of Engineering Sciences

Sheet 4 (home)

Exercise 1:

Consider the linear system of differential equations

$$\boldsymbol{y}' = \underbrace{\begin{pmatrix} 3 & 4 \\ 4 & -3 \end{pmatrix}}_{=: \boldsymbol{A}} \boldsymbol{y} \ .$$

a) Show by induction that

$$\boldsymbol{A}^k = 5^{k-1} \begin{pmatrix} 4 & 2 \\ 2 & 1 \end{pmatrix} + (-5)^{k-1} \begin{pmatrix} -1 & 2 \\ 2 & -4 \end{pmatrix}$$

for $k \in \mathbb{N}$.

b) Compute the matrix exponential solution $e^{x\boldsymbol{A}}$ of the system.

c) Compute the fundamental system by means of eigenvalues and eigenvectors of $\boldsymbol{A}$ and compare the result with b) .

Exercise 2:

Consider the differential equation

$$y'' + \frac{4}{x} y' - \frac{4}{x^2} y = -18 \, .$$

a) Determine a fundamental system by the method of reduction.

Hint: There is a polynomial solution $u(x) = ax + b$.

b) Rewrite the differential equation as a system of first order and compute a particular solution of the inhomogeneous equation by variation of constants.

c) State the general solution of the differential equation.

Hand in until: 10.12.2021