Differential Equations I for Students of Engineering Sciences

Sheet 4 (in-class)

Exercise 1:

a) Determine the general real-valued solution of the homogeneous system of differential equations
$$\boldsymbol{y}' = \begin{pmatrix} 1 & 1 \\ -2 & -1 \end{pmatrix} \boldsymbol{y} \ .$$

b) Consider the initial value problem
$$\boldsymbol{y}' = \begin{pmatrix} -2 & 1 \\ 3 & -4 \end{pmatrix} \boldsymbol{y} + \begin{pmatrix} 3 \\ -2 \end{pmatrix}, \quad \boldsymbol{y}(0) = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \ .$$

   (i) Determine the general solution of the homogeneous system.
   (ii) Compute a particular solution of the inhomogeneous system by variation of constants and alternatively using the ansatz $\boldsymbol{y}_p(x) = \boldsymbol{a}$ with $\boldsymbol{a} \in \mathbb{R}^2$.
   (iii) Solve the initial value problem.

Exercise 2:

a) Consider the differential equation $y''' - 4y'' - 20y' + 48y = 0$

   (i) compute the general real-valued solution,
   (ii) rewrite the differential equation as a system of first order and
   (iii) compute eigenvalues, eigenvectors and a fundamental matrix of the system.

b) Compute the general real-valued solution for the following differential equations:

   (i) $y''' - y'' - 15y' - 25y = 0$,
   (ii) $y'''' - 4y''' - 2y'' + 12y' + 9y = 0$.

Date: 6.12. - 10.12.2021