Fachbereich Mathematik der Universität Hamburg **Winter 2021/22**
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Differential Equations I for Students of Engineering Sciences

Sheet 5 (in-class)

Exercise 1:

a) Solve the initial value problem

$$\ddot{y} - 6\dot{y} + 5y = 0, \quad y(0) = 0,\ \dot{y}(0) = 1$$

(i) by means of the characteristic polynomial and

(ii) by means of the Laplace Transformation.

b) Solve the initial value problem

$$\begin{aligned} u' &= 2u + v\,, \quad u(0) = 5 \\ v' &= 2v - u\,, \quad v(0) = 1 \end{aligned}$$

by means of the Laplace Transformation.

Hint: Let $\boldsymbol{A}$ be invertible with $a, b, c, d \in \mathbb{R}$. Then it holds that

$$\boldsymbol{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad \Rightarrow \quad \boldsymbol{A}^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}.$$

Exercise 2:

Solve the initial value problem

$$y'' + 4y = 0, \quad y(0) = 0, \quad y'(0) = 2$$

by means of a power series of the form $y(x) = \sum_{k=0}^{\infty} a_k x^k$.

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