Differential Equations I for Students of Engineering Sciences

Sheet 6 (in-class)

Exercise 1:

Compute the general solution of the differential equation

$$y'' + 16y = 16 \quad \text{with} \quad 0 \le x \le \frac{\pi}{8}.$$

Employ the general representation of the solution of the individual equation to compute all solutions for the following boundary conditions:

a) $4y(0) + y'\left(\frac{\pi}{8}\right) = 4$ and $y\left(\frac{\pi}{8}\right) = 0$,

b) $y(0) = 1$ and $y'\left(\frac{\pi}{8}\right) = 1$,

c) $y(0) + y'(0) = 0$ and $y\left(\frac{\pi}{8}\right) + y'\left(\frac{\pi}{8}\right) = 0$.

Exercise 2:

a) Compute the equilibrium of the following system of differential equations, investigate its stability, determine its type, compute the general solution of the system of differential equations and sketch the phase plot:

$$\begin{aligned}
\dot{x} &= 9x + 2y + 24, \\
\dot{y} &= 2x + 6y + 22.
\end{aligned}$$

b) Determine all stationary solutions (equilibria) of the system of differential equations

$$\begin{aligned}
\dot{x} &= 2y - xy \\
\dot{y} &= x - xy
\end{aligned}$$

and investigate their stability properties by (local) classification.

Exercise 3:

Compute the eigenvalues and eigenfunctions of the following Sturm-Liouville boundary eigenvalue problem

$$y'' - 2y + \lambda y = 0 \quad \text{with} \quad y'(0) = 0 \quad \text{and} \quad y(1) = 0 \,.$$

Date: 17.1. - 21.1.2022