Mathematics Department
Prof. Dr. J. Behrens

SoSe 2022

Mathematics III Exam
(Module: Differential Equations I)

September 6, 2022

Please mark each page with your name and your matriculation number.

Please write your surname, first name and matriculation number in block letters in the designated fields following. These entries will be stored.

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I was instructed about the fact that the exam performance will only be assessed if the Central Examination Office of TUHH verifies my official admission before the exam's beginning in retrospect.

(Signature)

Exercise	**Points**	**Evaluater**
1		
2		
3		
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5		

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Exercise 1: (1+4 points)

a) Compute the general solution of the following differential equation by using separation of variables

$$xy' - 3y = 0.$$

b) Solve the following initial value problem for the given Bernoulli differential equation

$$y' - y + 2y^2 = 0 \quad \text{and} \quad y(0) = \frac{1}{3}.$$

Exercise 2: (3 points)

Compute the general solution of the following system of differential equations

$$\boldsymbol{y}' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \boldsymbol{y} \; .$$

Hint: Note that $(1-\lambda)^2 - 1 = \lambda(\lambda - 2)$.

Exercise 3: (4 points)

Solve the following initial value problem

$$y'' + y' - 2y = e^x \,, \quad y(0) = 1 \,,\ y'(0) = \frac{4}{3} \,.$$

Exercise 4: (3 points)

Consider the linear differential equation

$$y'' + y = 0.$$

a) State a complex-valued fundamental system,

b) compute the real-valued general solution and

c) obtain all solutions of the corresponding boundary value problem with boundary values $y(0) = 2$ and $y(\pi) = -2$.

Exercise 5: (5 points)

Consider the following system of linear first-order differential equations:

$$\begin{aligned} \dot{x} &= x + 2y - 4 \\ \dot{y} &= 2x + y - 5\,. \end{aligned}$$

a) State the system in matrix-vector notation ,

b) compute all stationary solutions (equilibria),

c) and determine their stability properties.

d) Compute the general solution of the system of linear differential equations.