Department of Mathematics
Prof. Dr. J. Behrens

SuSe 2023

Mathematics III Exam
(Module: Differential Equations I)

05.09.2023

Please mark each page with your name and your matriculation number.

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I was instructed about the fact that the exam performance will only be assessed if the Central Examination Office of TUHH verifies my official admission before the exam's beginning in retrospect.

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Exercise	Points	Evaluator
1		
2		
3		
4		
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$\sum =$

Exercise 1 (6 points)

Determine the general solution of the differential equation

$$y'(t) + 2y(t) - t\,y(t)^4 = 0.$$

Exercise 2 (3 points)

Rewrite the following initial value problem as equivalent initial value problem for a system of first-order differential equations

$$y'''(x) - y''(x) + 2y'(x) - 3y(x) = 0, \qquad y(1) = 1,\ y'(1) = 4,\ y''(1) = 9.$$

Exercise 3: (5 points)

Consider the boundary value problem

$$
\begin{aligned}
y'' - 4y' + 4y &= h(x) \qquad x \in]0,1[\\
\alpha y(0) - y'(0) &= \gamma_1 \\
y(1) &= \gamma_2 \qquad \alpha, \gamma_1, \gamma_2 \in \mathbb{R}.
\end{aligned}
$$

For which values of α is the boundary problem uniquely solvable for any $\gamma_1, \gamma_2 \in \mathbb{R}$ and any continuous function $h(x)$ on the interval $[0,1]$?

Exercise 4: (2 points)

Consider the system of differential equations

$$\dot{\boldsymbol{y}}(t) = \begin{pmatrix} 0 & 1 \\ -\frac{1}{t^2} & \frac{3}{2t} \end{pmatrix} \boldsymbol{y}(t) + \begin{pmatrix} t^3 \\ 2t^2 \end{pmatrix}, \qquad t \geq 1.$$

The functions

$$\boldsymbol{y}^{[1]}(t) = \begin{pmatrix} 2\sqrt{t} \\ \frac{1}{\sqrt{t}} \end{pmatrix} \text{ and } \boldsymbol{y}^{[2]}(t) = \begin{pmatrix} t^2 \\ 2t \end{pmatrix}$$

are solutions of the corresponding homogeneous system of differential equations.

Do $\boldsymbol{y}^{[1]}$ and $\boldsymbol{y}^{[2]}$ build a fundamental system for the space of solutions of the corresponding homogeneous system of differential equations?

Exercise 5: (4 points)

Consider the initial value problem

$$y''(t) + 4y'(t) + 3y(t) = 2\cos(t) + t^2 e^{-2t}, \text{ for } t > 0, \qquad y(0) = 0, y'(0) = 5.$$

Into which algebraic equation can the initial value problem be transformed by the Laplace transformation?