# Mathematics III Exam (Module: Differential Equations I)

#### 26 August 2024

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Assessment according to examin. reg: with Analysis III single scoring

I was instructed about the fact that the exam performance will only be assessed if the Central Examination Office of TUHH verifies my official admission before the exams beginning in retrospect.

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Exercise	Points	Evaluator
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## Exercise 1) (3 points)

Compute the general solution of the following differential equation

$$y'(t) = \cos(t) \cdot \frac{1}{4y^2(t)}$$

#### Exercise 2) (5 points)

- a) Which of the following differential equations for u(t) is exact?
  - (i)  $u + u^3 + 3u^2u' = 0$ .
  - (ii)  $u^5 + \sin(t) + 5tu^4u' = 0$ .
  - (iii)  $ut^2 tu^2u' = 0$ .

#### Justify your answers.

b) Determine the corresponding potential and the general solution for an exact differential equation in part a).

## Exercise 3) (6 points)

Determine the general solution of the following differential equation

$$u'''(t) + 4u''(t) - 5u'(t) = -1 - 5t.$$

### Exercise 4) (6 points)

Consider the system of differential equations

$$\boldsymbol{u}'(t) = \boldsymbol{A} \cdot \boldsymbol{u}(t) = \begin{pmatrix} -1 & 0 & 0\\ 1 & 1 & \beta\\ 2 & -\beta & 1 \end{pmatrix} \cdot \boldsymbol{u}(t)$$

with parameter  $\beta \in \mathbb{R}$ .

- a) Analyse the stability of the stationary point  $(0,0,0)^T$  of the system.
- b) Let  $\beta = 0$ . Determine a fundamental system of the system of differential equations.