

## Differential Equations II for Engineering Students

### Work sheet 2

**Exercise 1** (Exam SuSe17, Ex.2a, Hinze/Kiani)

Compute the solution to the following initial value problem for  $u(x, t)$ :

$$\begin{aligned}u_t + \frac{1}{2}u_x &= -4u, & x \in \mathbb{R}, t \in \mathbb{R}^+, \\u(x, 0) &= 2 \sin(x) & x \in \mathbb{R}.\end{aligned}$$

**Exercise 2:**

Determine the solution  $u(x, y)$  to the following Differentialgleichung

$$xu_x + \frac{y}{2}u_y = u,$$

that satisfies the condition  $u(1, y) = 1 + y^2$ ,  $y \in \mathbb{R}$ .

**Exercise 3:** (only for people who compute fast)

Given the following initial value problem

$$\begin{aligned}u_t + 3u \cdot u_x &= 0, & x \in \mathbb{R}, t \in \mathbb{R}^+ \\u(x, 0) &= \begin{cases} 0 & \forall x \leq 0 \\ \frac{1}{3} & \forall x > 0 \end{cases}\end{aligned}$$

- Write down the system of characteristic equations.
- Are the characteristics straight lines?
- Draw the characteristics through the points  $(x_k, 0) := (k, 0)$  for  $k \in \{-3, -2, -1, 0, 1, 2, 3\}$ .  
Compute the values of the solution along these characteristics.
- Using parts a)-c), can you obtain the values of  $u(x, t)$  in the points  $(-1, 2)$ ,  $(1, 2)$  and  $(3, 2)$ ?

**Discussion: 25.04.-29.04.2022**