Fachbereich Mathematik der Universität Hamburg **SuSe 2022**
Prof. Dr. J. Behrens
Dr. H. P. Kiani
Sofiya Onyshkevych

Differential Equations II for Engineering Students

Work sheet 2

Exercise 1:

Determine the entropy solution to Burgers' equation $u_t + uu_x = 0$ with the initial data

a) $u(x,0) = \begin{cases} 0 & x < 0 \\ 1 & 0 \le x \le 1 \\ 2 & 1 < x \end{cases}$ and **b)** $u(x,0) = \begin{cases} 2 & x < 0 \\ 1 & 0 \le x \le 2 \\ 0 & 2 < x \end{cases}$

Exercise 2)

Given a conservation equation $u_t + \left(\frac{u^4}{16}\right)_x = 0, \qquad x \in \mathbb{R},\, t \in \mathbb{R}^+ .$

a) Are the characteristics $(x(t), t)$ straight lines? Explain your answer.

b) Given the initial data $u(x,0) = 2 + \arctan(x)$, $x \in \mathbb{R}$. Determine the characteristic through the point $(0,0)$.

c) Check which of the functions $u^*, \tilde{u}, \hat{u}$ given below is the (weak) entropy solution for the initial values

$$u(x,0) = \begin{cases} 2 & \text{for } x \le 0, \\ 1 & \text{for } x > 0 \end{cases}$$

$$u^*(x,t) = \begin{cases} 2 & \text{for } x \le \frac{3}{2}t, \\ 1 & \text{for } x > \frac{3}{2}t. \end{cases} \qquad \tilde{u}(x,t) = \begin{cases} 2 & \text{for } x \le \frac{15}{16}t, \\ 1 & \text{for } x > \frac{15}{16}t. \end{cases}$$

$$\hat{u}(x,t) = \begin{cases} 2 & \text{for } x \le 0, \\ 2 - \frac{x}{t} & \text{for } 0 < x \le t, \\ 1 & \text{for } x > t. \end{cases}$$

Discussion: 09.05.-13.05.2022