Fachbereich Mathematik der Universität Hamburg **SuSe 2022**
Prof. Dr. J. Behrens
Dr. H. P. Kiani, S. Onyshkevych

Differential Equations II for Engineering Students

Work sheet 4

Exercise 1:

Determine the type of the following differential equations

a) $u_{xx} + 4u_{xt} - 5u_{tt} = 0,$

b) $10u_{xx} + 6u_{xy} + u_{yy} = 0$

c) $4x^2\, u_{xx} + 8xy\, u_{xy} + y^2\, u_{yy} + 2x\, u_x = 0$

Exercise 2:
Determine all rotationally symmetrical solutions of the following boundary value problem

$$\begin{aligned} \Delta u &= -\frac{1}{\sqrt{x^2+y^2}} \quad \text{for } 1 < x^2+y^2 < 9, \\ u(x,y) &= 1 \quad \text{on } x^2+y^2 = 1, \\ u(x,y) &= 2 \quad \text{on } x^2+y^2 = 9. \end{aligned}$$

Note: Laplace equation in polar coordinates is given by $u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\varphi\varphi} = 0$.
Rotationally symmetrical means that it does not depend on ϕ .

Exercise 3: Only for very fast students

a) Let u be the solution of the boundary value problem

$$\begin{aligned} \Delta u &= -1 \qquad |x| < 1, |y| < 1, \\ u(x,y) &= 0 \qquad |x| = 1 \text{ or } |y| = 1 \end{aligned}$$

and $v(x,y) = u(x,y) + \frac{1}{4}(x^2+y^2)$.

Show that $v(x,y)$ solves Laplace equation, and determine the upper and lower bounds for $u(0,0)$.

b) Let $u(x,y)$ is a solution to the problem:

$$\begin{aligned} \Delta u &= 0, \qquad \text{in } \Omega :=]0,2[\times]0,1[\\ u(x,y) &= 3x^2 \qquad\qquad \text{on } \partial\Omega\,. \end{aligned}$$

Determine, without computing u, for each of the following statements whether it is true. Explain your answers.

- It holds $\max_{(x,y)\in\bar{\Omega}} u(x,y) = 2$.
- It holds $\min_{(x,y)\in\bar{\Omega}} u(x,y) = 0$.
- $u(x,y) = 3x^2 - 3y^2$ is a solution to the boundary value problem.

Discussion: 30.05.-03.06.2022