

Differential Equations II for Engineering Students

Homework sheet 5

Exercise 1:

a) Derive Green's function for a sphere of radius R around the origin.

b) Let

\bar{x} =reflection of $x = (x_1, x_2)$ through the x_1 -axis,

\hat{x} =reflection of x through the x_2 -axis,

\tilde{x} =reflection of x through the origin

Show that the Green's function for the quadrant $x_1 > 0, x_2 > 0$ can be constructed using the correction function

$$\Phi^x(y) := \Phi(y - \bar{x}) + \Phi(y - \hat{x}) - \Phi(y - \tilde{x})$$

Exercise 2

Determine the solutions to the following tasks using the suitable product ansatz.

a)

$$\begin{aligned} u_t &= u_{xx} & x \in \mathbb{R}, t \in \mathbb{R}^+, \\ u(x, 0) &= \sin(x) + 2 \cos(2x) & x \in \mathbb{R}. \end{aligned}$$

b)

$$\begin{aligned} u_t - u_{xx} &= 0 & 0 < x < \pi, t \in \mathbb{R}^+, \\ u(x, 0) &= \frac{\sin(2x)}{2} + \frac{\sin(4x)}{4} & 0 < x < \pi \\ u(\pi, t) &= u(0, t) = 0 & t > 0. \end{aligned}$$